INCREASED STUDENT PARTICIPATION IN A DISCRETE MATHEMATICS COURSE*

Suzanne Fox Buchele
Department of Mathematics and Computer Science
Southwestern University
Georgetown, Texas 78626
(512) 863-1361
bucheles@southwestern.edu

ABSTRACT

A new approach to structuring a Discrete Mathematics course is discussed. The approach centers on student presentations of homework problems during class. Students are encouraged to work collaboratively on homework, but to write up their solutions and/or proofs on their own. Individual students are required to present problems to the class regularly, and approximately 1/2 of each class session is spent going through the homework via student-led presentations. However, homework is weighted relatively low in the course grade, while exams are weighted more heavily. Therefore, the overall approach to the class is for the students to master the material in order to be successful on the individual exams, while encouraging collegial collaborative work inside and outside of class. The course in question was taught using this method five times over five years at a small liberal arts college. Advantages of the approach are discussed, including student engagement in the class, peer modeling of successful mastery of course material, and an improvement of communication skills.

INTRODUCTION AND MOTIVATION

Discrete Mathematics classes must cover a lot of material. For example, if one is following the Curriculum 2001 guidelines [3], the topics must include: functions, sets, relations, basic logic, proof techniques, basics of counting, graphs and trees, and discrete probability. Although the Curriculum 2001 guidelines indicate that these core areas can

---

be covered in the time allotted for a typical one-semester course, it is difficult to cover the breadth at a depth sufficient for true student understanding of and engagement in the topics. For most of these topic areas, student understanding of the material is facilitated by the student engaging multiple problems in the topic area. However, this typically means that either the professor grading workload is high (and, homework may not be graded in a timely fashion), or, not all homework is graded. Both of these consequences are problematic. A high grading workload is not desirable due to the high workload many computer science educators already face. For example, computer science professors often encounter various unpredictable technology-related problems that cause something that worked last semester to not work this semester (or, something that worked last week to not work this week). In addition, keeping up with the changing technologies inherent in the field of computer science is time consuming. Lastly, grading programs can be a difficult and time-consuming endeavor for computer science professors as well. With a high-grading workload, it is common for homework to be returned more than one day after it is handed in by the student, due to a grading backlog the professor may face. This is particularly problematic in a fast-paced course such as Discrete Mathematics, in which it is important for the student to get appropriate feedback on their work in a timely fashion, before moving on to the next topic. However, not grading all homework is also not desirable, since the student is missing valuable feedback on work that s/he may think is correct, but in fact is not. Therefore, a different approach to incorporating significant student work in a discrete mathematics course is desirable.

DISCRETE MATHEMATICS COURSE DESCRIPTION

The Discrete Mathematics course in question [1] is taught at a small liberal arts college in a joint mathematics and computer science department. Computer science faculty with substantial mathematics background, or, faculty who teach both mathematics and computer science, teach the class. Over the last five years, on average 12 students registered for the course and approximately 10 students completed the class, which was taught once per year. The largest class size was 18 students initially registered for the course, and the smallest class size was 6 students completing the class. The course content evolved slightly over the five years, but by the fifth year all the Curriculum 2001 topics were covered in the class except spanning trees and tree traversal strategies, which are covered in a data structures class. Because of the breadth of material covered and the depth necessary to ensure student understanding, the pace of the class is fairly quick. Emphasis in the class is on proof and proof techniques. The prerequisites for the class are Calculus II and CS II, although in practice the prerequisites could be reduced to Calculus I and CS I, and may in fact be reduced in the future. The class is run very much like a mathematics course, with examples and discussion of applications related to computer science (for example, students are asked to apply induction proof strategies to prove the correctness of loops given in pseudo-code). All work in the class is paper-pencil work.

NEW APPROACH TO DISCRETE MATHEMATICS COURSE

Overview of Approach

This approach to the Discrete Mathematics course involves increasing student involvement in the course. While the course is not taught using the Modified Moore Method [2], there are common elements between the Discrete Mathematics course and a modified Moore Method course. For example, all substantial homework problems are presented to the board by students, and mistakes are pointed out by the class or professor at the time of the presentation. However, the class structure differs greatly from a Modified Moore Method course in that a textbook is used ([9]), students are encouraged to work with each other on homework, and mistakes made during proof presentations are fixed at the time of the presentation through a collaborative effort of the presenting student, the class, and the professor. Various elements of Cooperative Learning [6] have been used in computer science and information systems college classrooms with success (for example, [4], [5], [8], [12]). The Discrete Mathematics class structure does not fully implement the standard Cooperative Learning model, although there are common elements between the Cooperative Learning model and the Discrete Mathematics course as well. Students work together on homework problems in groups outside of class, and discuss homework problems in class as one large group. Students are individually accountable for their learning in their groups, since their actual homework scores are weighted significantly less than their individual exam grades in the computation of their course grade. Therefore, each student has a high level of motivation to contribute positively to the group and engage fully in the learning process. However, groups are student-formed, not necessarily heterogeneous, and no group process methodologies are employed for group process improvement. At a basic level, the Discrete Mathematics course as implemented is an example of the *conversational classroom* [11], by promoting engagement in the course material by promoting participation with peers inside and outside of the classroom.

Daily Class Structure

Homework is assigned every class period. Students are encouraged to work together in groups on the homework outside of class. At the beginning of every class period, approximately 1/2 of the class time is spent going over homework. To facilitate this, students are requested to present their homework problems to the class. The professor either collects the homework for professor-grading, or asks students to swap papers and homework is peer-graded based on the homework solutions presented on the board. The professor then covers new material for approximately 1/2 of the class time, new homework is assigned, and class is dismissed. If the homework was peer-graded, students are requested to report their grade to the professor as they leave the classroom.

Homework Presentations

The student-presented homework is the key to the method described here. Homework is assigned every class period (three times per week), in an amount that should take the average student approximately one to two hours, occasionally longer. All students are required to present homework problems to the class regularly, so getting students to volunteer for any given problem is typically not an issue. At the beginning of each class period, the professor asks for volunteers for each of the problems due that day. Some problems, particularly short computational or straightforward (non-proof) applications of a formula, are not student-presented but instead the professor will give the answers to these problems to the class. Of problems eligible to be presented, students volunteer for specific problems. All students who are volunteering that day then begin to write their solutions on the board, all at once. When all students have completed writing their solutions to the board, one student begins. S/he presents the problem to the class, describes the overall solution approach, including naming the proof technique, and then describes, step by step, his or her solution. Often, thought processes behind the major ideas of the solution are presented as well. If there are unresolved issues with the solution, another student may point out an error, or question a step or the entire methodology of the solution. If no student objects and there are remaining errors in the solution presented, the professor will step in and describe the issue. After receiving questions or issues regarding his or her solution, the presenting student must revise the problem solution on-the-fly. To ensure a non-threatening process, the professor will lead the student as little or as much as necessary in order for the student to successfully end up with a completely correct solution by the time s/he is done presenting. After each presentation, before the class moves on to the next problem, the professor asks the class if any other student has a significantly differing approach or solution. If so, the differing approach or solution methodology is discussed, and that student may be asked to present their different approach as well, if time allows. Then, the next presenting student proceeds with the next problem. This continues until all problems (typically three to five problems total) due that day are presented. During each presentation, non-presenting students are asked to self-correct their own homework based on the presentations, using a different type or color of writing utensil so that self-corrections can be easily identified. At that point, either the professor collects up the homework to be graded and returned to the students on the next class day, or, each student passes their paper to another student for peer-grading. The solutions are still on the board for the peer-grading process. The professor assigns points to each problem or sub-problem, and orally gives the class approximate number of points off for various common errors. Questions about how much to take off for certain errors are asked and answered immediately during the peer grading process. After papers are collected or peer-graded, the professor presents new material. After class, if the homework was peer-graded, the professor asks students to report their homework grade for the day to the professor as they are leaving class.

Homework is collected by the professor more often at the beginning of the semester and at least once per major topic area, typically 9-12 times over the course of a 15 week semester in a three day per week class. Homework is collected most class periods when the students are first learning and perfecting proof techniques early on in the course. Typically, self-grading and peer-grading are more lenient than professor-grading, and so it is necessary and important to collect the homework occasionally throughout the semester so that students have a realistic idea of how the professor will grade their solutions on exams.

Course Structure

Students are encouraged to work on homework in pairs or in small groups. However, it is required that students write up their solutions separately. This is to ensure that the students are learning the language of mathematics as they are working together on problem solving. Students are asked to ensure that each member of a study group contributes approximately equally to the group, although this is not proctored and often doesn't happen. In a small department, students tend to all know each other already, and so groups are generally formed easily and persist throughout the semester. Observations over the five years the course was taught echo research findings [7] that groups of three students tend to be the most successful group size, and students in groups of only two low-achieving students do not necessarily reap benefits from working together. To date, no requirements for homework group formation have been mandated by the professor, although suggestions are sometimes given. In order to ensure that students do not rely too heavily on other students, homework is given a relatively small weight of the overall course grade (10%), and student presentations are weighted similarly (5-10%). The bulk of the student's course grade is from exams, typically three in a semester, plus the comprehensive final exam. Therefore, students are motivated to learn the material well, using collaborative discussion and group work as much as possible, with the end goal of being able to individually solve similar problems on the (individual) exams. Students are required to keep their homework and produce it whenever asked by the professor, so that self-reported grades can be verified if suspicions arise. Because the homework presentations are such a public and integral part of the class, and a major method by which course material is learned, students are highly motivated to complete their homework and present it in class. Peer pressure has a positive impact in this regard. Furthermore, since the professor does not announce if s/he will collect the homework on a given day until after the homework presentations, students must always pay close attention to the homework presentations by their peers in order to self-grade their papers. Also, since students peer-grade other student's papers, they must be able to analyze what the other student did and reason whether or not it yielded a correct solution. The professor is available to the class during the peer-grading process, to help students if they have questions.

STUDENT RESPONSE TO NEW DISCRETE MATHEMATICS COURSE

The Discrete Mathematics course was taught using this method five times over five years to over 50 students. Students generally found the course rigorous and challenging. Much to the professor's surprise, student response to the homework presentations and overall course structure was overwhelmingly positive. An open-ended question on the student course evaluation asked what the student liked or found helpful about the course; between 25% and 50% of the students in each of the five classes specifically wrote that they liked having students present problems to the board in class, although that response was in no way specifically prompted for. Sample responses included, "I thought the homework presentations were good", "I thought the presentation of homework was very helpful and promoted greater understanding of the concepts", and "I liked that we opened with homework but that it didn't take up all the class time, and we learned new things".

Other responses included positive comments such as the class "was interactive" and promoted "student involvement in class".

About 20% of the students commented on the relative weighting of homework vs. exams, although approximately 10% would have preferred less emphasis on homework and approximately 10% would have preferred more. About 10% of the students commented that they disliked having daily homework (although over 5% commented that they found the daily homework helpful to keep up with the class topics). During the first 3 years, the classes were given a course evaluation with a question that asked students to rate, on a scale of 1 to 5, (with 1 being too little, 3 being just right, and 5 being too much), the amount of student participation in the class. About 75% of those students rated the amount of student participation as a 3 (just right), with about 25% of the students approximately equally split between a 2 (not quite enough) and a 4 (a bit too much). After the third year, the course evaluation form changed and there was not a similar question on the new form.

ADVANTAGES TO NEW DISCRETE MATHEMATICS COURSE

Aside from student reviews, there are perceived student outcome advantages of this approach to teaching the Discrete Mathematics course as well. There is great value in the student homework presentations, for several reasons. First, at a liberal arts college, we are charged with producing educated students with excellent communication skills. The frequent presentations certainly help with communication skills, including speaking, mathematical writing, oral logical reasoning, and simple blackboard skills. This professor can often see a marked improvement in presentation style over the course of the semester, both due to practice and from modeling by student peers. Another advantage to the homework presentations is that they are done by the students, for the students. For those students who have a fear of or aversion to mathematics, homework presentations done by faculty simply reinforce the notion that "yes, that professor is smart and knows how to do this because s/he has an advanced degree". Homework presentations by students are another matter. In the homework presentations, other students in the class are seen understanding, wrestling with, and being successful presenting difficult material to the class. Furthermore, when errors in student presentations arise (which they frequently do), students see the presenting student engaged enough in the material to alter their problem presentation on-the-fly. By witnessing other students successfully engaging the difficult course material, students see the level of involvement required to succeed in the class. Since the course is required for the computer science major, students decide early on in the course if they are willing to engage the class as fully as it requires, or drop the course and tackle it at another time when they are prepared to devote the time and energy necessary. The students in the class more readily recognize that the course material is hard, rather than "Professor X is hard". Students also pay attention in class more readily and are more engaged in the class sessions, because they often have to self-grade and then peer-grade another paper. This gives them more accountability in the class.

This approach to the Discrete Mathematics course is particularly useful in the teaching of proper proof technique. The basic proof techniques are covered early in the class, after a review of basic mathematics and set theory. Since the emphasis of the class is on proof and proof techniques, most student homework presentations involve proof.

The student is required to tell what type of proof they are attempting (direct or contrapositive, for example, or even proof by cases, with direct proof for the first case and contradiction for the other cases, etc.) and then are expected to follow the proof technique that they have outlined. This "sets up" the problem for both the presenting student and for the class to follow logically. In addition, since the professor is interacting with the presenting student, steps in the proof that are not as clear as they could be, or are more of a "stretch" than a single step in a proof ought to be, are made more precise and elaborated on as the student is presenting. Thus, the level of detail that is required for proofs is constantly being reinforced throughout the class, in a manner quite different from the professor simply presenting finished proofs at a detailed level. As a consequence, students learn proof techniques readily.

A further advantage to the homework presentations is the faculty engagement in the individual student's learning process. Each student is regularly presenting problems in class, including written and verbal presentation of solutions as well as a discussion of the logical reasoning behind each solution. The professor is engaged in this process as well, and often discusses the problem and/or the solution with the student in front of the class. This is especially done if there are errors in the solution presented, but also occurs as the professor connects the problem with other problems, other concepts, or other interesting material or applications associated with the problem. By engaging students in this way individually, the professor can more easily identify which students need help with certain concepts, and then invite them to office hours to target help specifically for them. However, for this advantage to be realized, small class sizes are important. During the five years the Discrete Mathematics course was taught in this way, the class size was never greater that 18 students, with an average class size of 10 students.

Another advantage to the course structure is the freedom of students to work together with other students. Students who are dubious of their own mathematical reasoning ability have the opportunity to work with other students in groups on the homework. Although it has been reported [10] that students tend to resist group work, experience with this Discrete Mathematics course has shown that students tend to prefer working on homework in groups.

Although this method leaves only half of a class period or less to present new material, the spirit of the course is "learning through doing"; basic definitions are introduced, handouts are given for some topics to clarify material given in the book or to give extra examples, and students are required to learn the material through doing the homework. Twenty minutes is typically plenty of time to introduce a topic well enough for students to launch into homework. In addition, it can easily be argued that students learn more from their engagement in the homework than they would by listening to a "talking head" presenting the material in class for another twenty to thirty minutes per class period.

Overall, this approach to structuring a Discrete Mathematics class decreases professor workload (as measured both by time preparing a "lecture" and in out-of-class grading), while it produces well-prepared students who are engaged with the material and who improve their communication skills. Professor workload is potentially increased in the one-on-one time spent working with students on their homework presentations in class, and targeting students to meet with one-on-one outside of class. However, many computer science students will always find mathematical topics difficult, and so one-on-one outside of class consultation with students will always occur. Working with students one-on-one at the board in class is time that would be spent in class on similar concepts anyway, either lecturing or doing another activity. The process of pointing out or leading a student to discover errors, and working through how to overcome these errors, is an invaluable form of teaching in which students get to see the thought process behind working through errors. Students also witness the thought process behind constructing proofs during the homework presentations as well. This type of teaching and learning is very valuable to the students, and though the professor must necessarily be fully engaged in the class and quick to evaluate problems and suggest alternatives, this is not necessarily any more difficult (and, one might argue far more enjoyable) than lecturing.

CONCLUSIONS

A new approach to a Discrete Mathematics course at a small liberal arts college is presented. Class sizes ranged from 6 to 18 students, although it is expected that class sizes up to 20 to 25 students could be accommodated using this technique. The course was taught in this manner five times over a five year period, with perceived positive results from both the professor and student perspective. The structure of the course facilitated student involvement in class and engagement in the material. Students were encouraged to work with each other on homework, and were able to see other students successfully struggle with and master class material during in-class homework presentations. The course structure ensured students were individually mastering the material via heavily weighted exams, was interactive and enjoyable to teach, and had a reduced grading workload due to the significant use of peer-grading in class.

REFERENCES

[1] Buchele, S., "Discrete Mathematics Syllabus", http://www.southwestern.edu/~bucheles/Discrete/syllabusf03.html, August 2003.

[2] Chalice, D.R., How to Teach a Class by the Modified Moore Method, *American Mathematical Monthly*, 102(4), pp. 317-320, 1995.

[3] Computing Curricula 2001: Computer Science Volume, Report of the ACM/IEEE-CS Joint Curriculum Taskforce, http://www. sigcse.org/cc2001, Dec. 15 2001,.

[4] Fellers, J.W., Teaching Teamwork: Exploring the Use of Cooperative Learning Teams in Information Systems Education, *The DATA BASE for Advances in Information Systems*, 27(2), pp. 44-60, 1996.

[5] Howard, R.A., Carver, C.A., Lane, W.D., Felder's Learning Styles, Bloom's Taxonomy, and the Kolb Learning Cycle: Tying it all Together in the CS 2 Course, ACM SIGCSE Bulletin, 28(1), pp. 227-231, 1996.

[6] Johnson, D.W., Johnson, R.T., Smith, K.A. *Active Learning: Cooperation in the College Classroom*, Edina, MN: Interaction Book Company, 1991.

[7] Ma, X., The Effects of Cooperative Homework on Mathematics Achievements of Chinese High School Students, *Educational Studies in Mathematics*, 31(4), pp. 379-387, 1996.

[8] Powers, K.D., A Collaborative Approach to Teaching Computer Architecture, *Journal of Computing in Small Colleges*, 16(1), pp. 69-80, 2000.

[9] Ross, K.A., and Wright, C.R.B., Discrete Mathematics (Fifth Edition), Upper Saddle River, NJ: Prentice Hall, 2002.

[10] Waite, W.M., Leonardi, P.M., Student Culture vs Group Work in Computer Science, *ACM SIGCSE Bulletin*, 36(1), pp. 12-16, 2004.

[11] Waite, W.M., Jackson, M.H., Diwan, D., The Conversational Classroom, *ACM SIGCSE Bulletin*, 35(1), pp. 127-131, 2003.

[12] Yerion, K.A., Rinehart, J.A., Guidelines for Collaborative Learning in Computer Science, *ACM SIGCSE Bulletin*, 27(4), pp. 29-34, 1995.